ELECTROMAGNETICALLY ENGENDERED CONVECTION IN ELECTROMAGNETOPHORESIS

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SUMMARY

Convection currents are engendered in a cell traversed by an electric current and a non-parallel magnetic field. In the case considered the fields are mutually perpendicular and at least one of the two fields is non-homogeneous. The convection engendered by non-uniformities in the fields employed results from non-uniformities in the electromagnetic force density and is analogous to thermal convection in a gravitational field.

An expression is derived for average speed of convection. The ratio of this speed to the speed of electromagnetophoretic migration of a small sphere of arbitrary radius is determined. The measure of precision is given for the dimensions of a migration cell which must be maintained to avoid masking the migration effect by convection in an unstabilized liquid column. The possibility of compensating for non-uniformities in the current distribution by introducing a non-uniformity in the magnetic field is discussed.

INTRODUCTION

Electromagnetophoresis[1] is a physical phenomenon which is of interest primarily in connection with solutions of biochemical and biophysical problems. It is, however, as yet not sufficiently widely known and adequately developed to have found many applications. The reason for its potential usefulness in these fields is as follows: It utilizes a physical property of suspended particles for their separation which, for biological suspensions, varies over a much wider range than electrical mobility in common electrophoretic separations or the density in centrifugal fractionations of biological materials. The property utilized for electromagnetophoretic fractionation of suspensions of cells and cell fragments is their electrical conductivity which may vary over several orders of magnitude between different species of suspended particles of biological interest. In addition, by using alternating magnetic fields combined with alternating electric currents, electromagnetophoresis offers the possibility of distinguishing suspended cells on the basis of differences in the conductivity of protoplasm, regardless of the differences in the permeabilities of the cell membranes (at

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high frequencies) or (at low frequencies) of fractionating cell suspensions on the basis of differences in permeabilities of the cell membranes.

In a similar way as electrophoresis, electromagnetophoresis is subject to disturbances by fluid convection. In addition to thermal convection, there is a second type of convection which may be disturbing in electromagnetophoresis; it is electromagnetically engendered convection. A method for elimination of the former type of convection has been suggested in a previous publication[2]. The aim of the present considerations is to obtain an approximate estimate of the magnitude of the latter type of convection to be expected in practice.

Electromagnetophoresis[1, 3–5] consists of migration of charged or neutral particles suspended in an electrolyte of an electrical conductivity differing from theirs. The forces causing this migration are produced by maintaining a current of density $\boldsymbol{J}$ and a magnetic field of flux density $\mathbf{B}$, both of them being in phase, in a suitably shaped cell. In the simplest case, $\boldsymbol{J}$ and $\mathbf{B}$ are uniform throughout the cell. But the

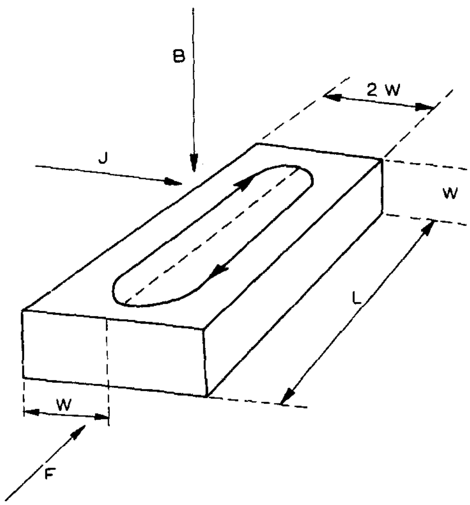

Fig. 1. Electromagnetophoresis cell.

condition of perfect uniformity can never be strictly obtained. A deviation from uniformity of $[\boldsymbol{J} \times \boldsymbol{B}]$ may lead to circulation of the electrolyte in the cell in cases where curl $[\boldsymbol{J} \times \mathbf{B}] \neq 0$, where $[\boldsymbol{J} \times \boldsymbol{B}] = \boldsymbol{f}$ is the force per unit volume of the electrolyte.

Circulation of the type illustrated in Fig. 1 is engendered due to variation of $\boldsymbol{J}$ or/and $\mathbf{B}$ at right angles to the force axis (indicated by vector $\mathbf{F}$ in Fig. 1). An increase in $[\boldsymbol{J} \times \mathbf{B}]$ in the direction of the **F**-axis does not give rise to circulation. This situation is analogous to thermal convection where horizontal density gradients may lead to thermal convection, whereas an increase in density in the direction of gravitational acceleration does not.

In order to estimate the rate of convection engendered in a cell of the shape represented in Fig. 1, we imagine a septum, represented by the dashed line, to divide our cell longitudinally into two equal parts. The septum is shorter than the length L so that circulation of the fluid is possible in the manner indicated by the closed

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stream line marked by two arrowheads. We treat the flow of the liquid as a Poisoeille flow in a pipe of cross-sectional area W^2 and approximate length $2L$. We treat this pipe in our computation as a round one of radius ϱ such that $\pi\varrho^2 = W^2$. We obtain then for the average speed of convection:

$$\bar{v}_c = \frac{\varrho^2}{8\eta}\frac{dp}{dL} = \frac{\varrho^2}{8\eta}\frac{\Delta f}{2} = \frac{W^2}{8\pi\eta}\frac{\Delta f}{2} \tag{1}$$

where $\boldsymbol{f} = [\boldsymbol{J} \times \mathbf{B}]$ and $\Delta \boldsymbol{f} = [\boldsymbol{J}_1 \times \mathbf{B}_1] - [\boldsymbol{J}_2 \times \mathbf{B}_2]$. $\boldsymbol{J}_1$ and $\mathbf{B}_1$, are the average values of $\boldsymbol{J}$ and $\mathbf{B}$ in the channels of the cell to the left of the septum, whereas $\boldsymbol{J}_2$ and $\mathbf{B}_2$ are the corresponding values for the channel to the right of the septum. $\Delta \boldsymbol{f}$ is a measure of the transverse variation of $\boldsymbol{f} = [\boldsymbol{J} \times \mathbf{B}]$ across the cell.

The electromagnetophoretic velocity of a small sphere of a conductivity greatly superior to that of the surrounding fluid is[2,3]:

$$v_s = \frac{JBr^2}{3\eta}. \tag{2}$$

The ratio of the speed of convection to the speed of migration is

$$\frac{\bar{v}_c}{v_s} = \frac{3}{16\pi}\frac{\Delta f}{f}\left(\frac{W}{r}\right)^2 \simeq \frac{1}{4}\frac{\Delta f}{f}\left(\frac{W}{d}\right)^2 \tag{3}$$

(where $d = 2r$ is the diameter of the spherical particle).

Magnetic fields of great uniformity can be obtained which will vary but negligibly over a distance of a few millimeters, which is a suitable order of magnitude for the width W of an electromagnetophoresis cell. Assuming $\mathbf{B} = \text{const.}$, $\Delta \boldsymbol{f}$ becomes dependent on the precision with which the cell can be made since then $\Delta f/f = \Delta J/J$ and $J = i/W^2$. If ΔW designates the lateral variation in W, we have

$$\frac{\Delta f}{f} = \frac{\Delta J}{J} = 2\frac{\Delta W}{W}. \tag{4}$$

In order to estimate the measure of precision $\Delta W/W$ which must be maintained in electromagnetophoresis cells in order to avoid masking the migration effect by the convection effect, we assume as tolerance for the convection velocity $\bar{v}_c = v_s$. Taking this and eqn. (4) into account, we obtain from eqn. (3):

$$1 \simeq \frac{1}{2}\frac{\Delta W}{W}\cdot\left(\frac{W}{d}\right)^2 \tag{5a}$$

and

$$\frac{\Delta W}{W} \simeq 2\left(\frac{d}{W}\right)^2. \tag{5b}$$

Actually the precision does not have to be as great as demanded by eqn. (5b). A circulation engendered by a non-uniformity in W, and hence in $\boldsymbol{J}$, can be compensated by a circulation in the opposite sense of rotation caused by a non-uniformity in $\mathbf{B}$, such that $\Delta \boldsymbol{f} = [\boldsymbol{J}_1 \times \mathbf{B}_1] - [\boldsymbol{J}_2 \times \mathbf{B}_2] = 0$. To achieve this balance, a somewhat non-uniform magnetic field must be used and the position of the cell must be adjusted in it so as to minimize the electromagnetically engendered convection. Under such conditions the precision requirement is given by

$$\frac{\Delta W}{W} = K\left(\frac{d}{W}\right)^2, \tag{6}$$

where the value of $K > 2$ depends on the effectiveness of the compensating adjustment.

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Another means of minimizing the required precision would be to make the ratio W/d as small as possible.

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REFERENCES

[1] A. KOLIN, *Science*, 117 (1953) 134.
[2] A. KOLIN, *J. Appl. Phys.*, 25 (1954) 1442.
[3] D. LEENOV AND A. KOLIN, *J. Chem. Phys.*, 22 (1954) 683.
[4] A. KOLIN, *J. Appl. Phys.*, 25 (1954) 1065.
[5] A. KOLIN AND R. T. KADO, *Nature*, 182, 1958, 510.

SUPPRESSION OF THERMAL CONVECTION AND OF SIMILAR TYPES OF FLUID STREAMING BY AN ELECTROMAGNETIC FORCE FIELD

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SUMMARY

A method is described for increasing the stability of a fluid column against convection (such as thermal or electromagnetic convection) by superimposing upon a stabilizing vertical density gradient a parallel gradient of electromagnetic force density. The latter gradient is generated by creating a gradient of electrical conductivity concomitant with the vertical density gradient and maintaining at right angles to this gradient electric current traversed by a perpendicular homogeneous magnetic field. To obtain a stable column, one must choose the directions of the current and the magnetic field such as to make the electromagnetic force vector point in the direction of increasing electrical conductivity. Conditions under which the stability of a density gradient is reinforced, diminished or neutralized are considered as well as the engendering of instability by electromagnetic forces in a column stabilized by a density gradient.

INTRODUCTION

Thermal convection is one of the main experimental difficulties in electrophoresis in liquid columns[1] and in ultracentrifugation[2]. It is equally disturbing in electromagneto-

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